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MEASUREMENT OF SELF-INDUCTANCE.

By H. D. HILL.

DISSERTATION

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VITA

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ART. IX. — *Measurement of Self-Inductance*; by J. B. WHITEHEAD and H. D. HILL.

THE absolute measurement and comparison of self-inductance has within the last few years become a problem of increasing importance. The best known method and that most generally used is the Rayleigh method. However, this necessitates very intelligent and careful work to obtain results concordant to one per cent. What seem to be the most accurate methods are various modifications of the Wheatstone bridge using alternating currents with an electro-dynamometer, telephone, or optical telephone as the measuring instrument.

In January, 1898, Professor Rowland* published a brief description of some twenty-six methods for the measurements and comparison of self-inductance capacity and mutual inductance. These methods are mostly of the Wheatstone bridge type and depend upon the use of an alternating current and an electro-dynamometer. As only a few of them have hitherto been tried, it was deemed desirable to systematically test them and determine their value, particularly in regard to the measurement of self-inductance. Such is the aim of the present investigation.

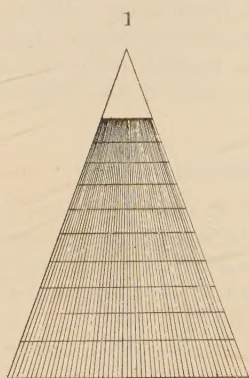
Source of current and frequency determination.

Since the value of the self-inductance or capacity as given by many of the methods depends upon the square of the current frequency, it was necessary to secure a current of harmonic wave form whose period was as constant as possible, and to devise a method of accurately determining that period at the moment of adjustment. This was very satisfactorily accomplished. In the deduction of the formulæ for all the methods the assumption is made that we have a simple harmonic electromotive force. In general an alternating current supplied by a dynamo has not only its fundamental period but also the odd upper partials. However, a generator constructed without too much iron and properly wound, if the field is not very strongly excited, will give a very good sine wave; especially if the resistance in the circuit contains self-inductance and no iron. These requirements were well satisfied by the dynamo used, which gave a very good sine wave. The alternator was constructed at the University. Its armature consisted of twelve coils fastened flat on a German silver plate revolving between twelve field pieces producing six poles. This dynamo was coupled directly to a direct current Crocker-Wheeler

* Phil. Mag. [5], xlv, 66-85, 1898.

motor. As long as the load on the dynamo was not varied the speed remained very constant. Greater constancy of speed was secured by operating the motor from storage batteries. The voltage furnished by the alternator could be controlled by slightly altering the strength of the field. For the determination of the frequency a chronograph was used. A kind of speed counter was connected to the shaft of the dynamo and a contact so arranged that in every ninety-seven revolutions of the dynamo armature a circuit was closed and a record made on the chronograph sheet. The chronograph was set up close to the rest of the apparatus and by means of a thread the pen could be raised for an instant from the sheet and so the exact time of making an adjustment could be readily noted. By interchanging the gear wheels the cylinder of the chronograph was made to run at about three and one-half times the customary speed. The distance between the checks indicating the seconds was so increased to over five and one-half centimeters.

A scale similar to that shown in fig. 1 was then constructed,



consisting of fine lines scratched on transparent celluloid and blackened with India ink. By means of this scale it was possible to read the chronograph sheet directly to fiftieths of a second and to estimate to five hundredths. However, in reading a chronograph sheet as exactly as this, certain precautions must be taken. First, the checks indicating the seconds should be sharp and distinct. Secondly, an error may arise if the chronograph does not run quite regularly. This can generally be remedied by properly adjusting the governor. By this method the frequency could be quite accurately determined, to 1/10 per cent or closer, if the sheet was read with sufficient care. It is to be noted that if sidereal seconds are recorded on the chronograph sheet they should be reduced to solar seconds, otherwise an error of 3/10 per cent is introduced in the absolute value of the frequency and, therefore, of 6/10 per cent in the value of self-inductance.

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Apparatus.

Electrodynamometer.—A Rowland electro-dynamometer was used. The self-inductance of the fixed coils was .0165 henry, and of the hanging coil .0007 henry.

Resistances.—In the measurements of self-inductance with alternating currents, ordinary resistance boxes are of little or

no value owing to the electrostatic capacity of the doubly wound coils. Except in the case of a few determinations with Method 25, all the resistances used were made of a special German silver resistance wire, B. and S. gauge, Nos. 30 and 27. This wire was not doubled, but wound continuously in one layer on slates or pieces of micanite, each slate containing about 2000 ohms conveniently subdivided. The self-inductance and capacity of such resistances is entirely negligible. For fine adjustment a small resistance box was constructed differing from an ordinary box in that its coils were wound flat on thin pieces of micanite.

Coils and Condenser.—The coils whose inductance were measured were as follows:

P, external diameter 33.46^{cm} , internal diameter 23.8^{cm} , was made of about 1200 turns of No. 16, B. and S. gauge single cotton-covered copper wire. Self-inductance as determined, $.5729$ henry. Resistance, 36.3 ohms.

C. Same dimensions as P except depth. It consisted of 1747 turns of No. 22 B. and S. gauge single cotton-covered copper wire. Self-inductance found to be 1.3025 henry. Resistance, 78 ohms.

S. External diameter 23.5^{cm} , internal diameter 15^{cm} , depth about 3.5^{cm} . It consisted of 2082 turns of No. 22 B. and S. single cotton-covered copper wire. Resistance, about 69 ohms. Self-inductance found to be 1.0331 henrys.

Condenser.—A $\frac{1}{2}$ microfarad mica condenser.

Conditions of sensibility.—The deflection of an electro-dynamometer is proportional to the product of the currents flowing in the fixed and hanging coils multiplied by the cosine of the phase difference of the two currents. In most of these Rowland methods the adjustment consists in so altering non-inductive resistances in one or more branches of the bridge that there is a ninety degree phase difference between the currents in the fixed and hanging coils respectively. Evidently in order to secure maximum sensibility the currents through the dynamometer coils should be as heavy as possible, heating alone being the limit.

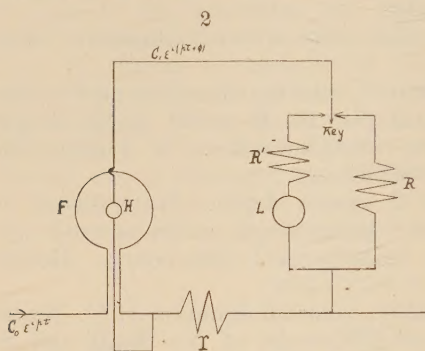
Sources of Error.—In work with alternating currents great care must be exercised that one part of the network does not exert an inductive action on another, e. g., that the coil whose inductance is being measured does not affect the hanging coil of the electro-dynamometer. Induction was carefully guarded against by the arrangement and tested for by means of reversing switches. The electrostatic capacity of doubly wound coils and resistances has already been mentioned as well as the precautions taken to avoid it. In the experiments here described all the connections were made as short as possible and no wires were twisted. Heating of the resistances was avoided as much

as possible and in the more accurate work the resistances of the several arms of the network were measured after adjustments on a Nalder standard "Post Office" box. All resistances are in International ohms.

Methods Used.

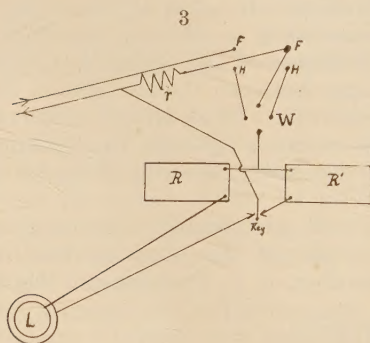
I.

Method 25.—The first method tried was that designated in Professor Rowland's article as number 25. It is an absolute method intended for the measurement of either self-induction



or capacity, the value of which depends upon the square of the current frequency. It was given a thorough test by Dr. T. D. Penniman,* except in so far as his method of measuring the frequency was rough compared with that used in this investigation, and he ascribes the lack of uniformity of his results chiefly to a want of

knowledge of the current period. It was therefore thought well to give the method a hasty trial in regard to the measurement of self-induction. In this method the hanging coil is shunted off the fixed coils circuit and the deflection with a non-inductive resistance in circuit with the hanging coil is



made the same as that of an inductive resistance in circuit with the hanging coil. The method and connections are shown in figs. 2 and 3; F and H represent the fixed and hanging coils of the electro-dynamometer; R, R' and r, the total resistances of the three branches; L is the self-inductance of the coil to be measured, W is a reversing commutator. The formula for the method

as deduced by Dr. Penniman is,

$$p^2 L^2 = (R' - R) (R + r)$$

where p is equal to 2π multiplied by the current frequency.

* This Journal, viii, p. 35, 1899.

This method did not prove satisfactory for measuring self-inductance. With care it is possible to obtain values which do not differ among themselves by more than 1 per cent. One drawback to the method is its lack of sensibility, that is, it is not usually possible to adjust the resistances closer than one part in three hundred. Again, since the electro-dynamometer was very "dead beat" it required a certain small time to compare the deflections, to be sure not long, but enough for the current period to alter slightly unless special precautions were taken. A few determinations of the self-inductance of coils S and C are given. It will be noticed that the values are lower than those obtained later by a more accurate method. This is probably due to the use of ordinary resistance boxes possessing electrostatic capacity. This method was the first tried and is the only one in which such resistances were used.

TABLE I. Coil C.

Zero.	Deflection.	R.	R'.	r.	n.	L.
24.98	15.30	149.7	1513.7	4.017	57.567	1.266 henrys
"	15.49	"	1531.7	"	57.227	1.282 "
24.82	15.18	"	1518.7	"	57.453	1.271 "
"	15.11	"	1530.7	"	57.510	1.275 "
24.77	12.70	199.7	1241.7	"	57.453	1.276 "
24.73	12.62	"	1238.7	"	57.510	1.273 "
						Mean 1.274 "

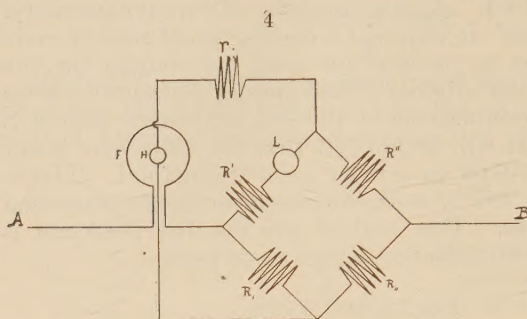
TABLE II. Coil S.

Zero.	Deflection.	r.	R.	R'	n (cycles).	L (henrys)
240.0	158.2	4.017	141.5	1300.7	63.885	1.023
"	158.5	"	"	1283.7	63.62	1.021
"	"	"	"	1283.7	63.329	1.020
239.2	301.0	"	"	1288.7	63.261	1.028
"	"	"	"	1281.7	"	1.025
239.0	134.2	"	191.5	1041.7	64.095	1.012
"	134.8	"	"	1038.7	63.955	1.013
238.7	345.8	"	"	1036.7	"	1.012
"	"	"	"	1038.7	"	1.013
249.0	121.3	"	241.5	920.7	64.095	1.014
"	"	"	"	922.7	"	1.015
239.1	129.9	"	291.5	850.7	64.167	1.009
"	"	"	"	"	64.095	1.010
239.7	"	"	491.5	836.7	64.23	1.025
						Mean 1.0171 henry

II.

Method 14.—The next method tested was that which in Professor Rowland's paper is number 14. It is an absolute method for measuring either self-inductance or capacity. It is a zero

deflection method depending upon adjusting the resistances until there is a ninety degree difference in phase between the currents flowing in the fixed and hanging coils of the electro-dynamometer. The method was devised and first used by Oberbeck,* later by Troje.† However, the frequency deter-



mination of both these investigators was quite crude and, as will be seen from the formula, the value of L or C varies as the square of the frequency. Diagrams of the method and connections are

given in figs. 4 and 5. As before, F and H represent the fixed and hanging coils of the electro-dynamometer; R_1, R_2, R_3, R_4 and r are the entire resistance of the several branches, L is the self-inductance of the coil to be measured, and l that of the hanging coil of the electro-dynamometer, W is a commutator for reversing the current through the hanging coil, S is a switch by means of which the current could be sent through a resistance X in value closely equal to the impedance of the network. The coil L , to avoid any possible effect on the rest of the network or hanging

coil of the electro-dynamometer, was placed some distance away usually on a level with the hanging and fixed coils and perpendicular to them.

If now an alternating electromotive force be applied to the terminals A and B , we may express the currents in the fixed and hanging coils of the electro-dynamometer by $C_0 e^{ipt}$ and $C_1 e^{i(p t + \phi)}$, p being equal to 2π times the frequency. By the application of Kirchhoff's laws to the Wheatstone bridge

* Wied. Ann., xvii, p. 816, 1882.

† Ibid., xvii, p. 501, 1892.

using "generalized resistances," we obtain as the ratio of the current in the battery arm to the current the bridge arm,

$$\frac{C_0}{C_i} \epsilon^{i\phi} = \frac{(r + ipl) (R_i + R + R' + R'' + ipL) (R_u + R'') (R_i + R' + ipL)}{R_i R'' - R_u R' - ipLR''}$$

Multiplying both sides of this expression by C_i^2 , rationalizing, and equating the real parts we obtain,

$$C_i \cos \phi = C_i^2 \frac{[R_i R'' - R_u R'] [r(R_i + R_u + R' + R'') - p^2 L l + [R' + R_i] [R_u + R'']]}{[R_i R'' - R_u R']^2 + p^2 L^2 R''^2}$$

$$C_i^2 \frac{p^2 L l R' (R_i + R_u + R' + R'') - p^2 L^2 r R_u - p^2 L^2 (R_u + R'') R_u}{R_i R'' - R_u R' + p^2 L^2 R''^2}$$

This expression must be equal to zero since we adjust the bridge until $\phi = 90^\circ$, therefore $\cos \phi = 0$. Equating to zero and simplifying,

$$L^2 + L l \left\{ \frac{(R'' + R_u) (R_i + R_u)}{R_u (r + R'' + R_u)} \right\} = \frac{R'' R_i - R_u R'}{p^2 R_u} \left\{ \frac{r(R_i + R_u + R' + R'') + (R' + R_i) (R'' + R_u)}{r + R'' + R_u} \right\}$$

This formula is somewhat simplified if R_u is chosen equal to R'' . However, for accurate work the equality must be exact. Since the self-inductance of the hanging coil of the Rowland electro-dynamometer is so small, only .0007 henry, the correction of L due to it is very small, in the case of coil C amounting to .0004 henry.

In using the method the numerical work involved in the calculation of L is rendered somewhat easier if we write the formula,

$$L + sLl = t$$

when very approximately, since l is so small,

$$L = t + \frac{S}{2} l$$

where

$$S = \frac{(R'' + R_u) (R_i + R_u)}{r + R_u + R''}$$

and,
$$t = \frac{1}{p^2} \left[R_i \frac{R''}{R_u} - R' \right] \left[R' + R_i + \frac{r(R_u + R'')}{r + R_u + R''} \right]$$

This method was found to be excellent. It is very sensitive and very accurate provided certain precautions are taken. Some little difficulty was experienced at first from the fact that the values of the self-inductance of a coil measured on any one day, although agreeing pretty well among themselves,

differed from those obtained at a different time. The cause of this variance proved to be the heating and consequent lack of knowledge of the resistances. In order to secure sensibility of the electro-dynamometer it was the custom to make the current through the fixed and hanging coils quite heavy. Sometimes the current through the fixed coils was nearly $\frac{1}{10}$ ampere. This was unnecessarily large, as with a current of $\frac{1}{20}$ ampere it is easily possible to adjust the resistance to one part in fifteen hundred or two thousand. If we wish to make exact measurements of inductance, the method requires an accurate knowledge of the resistances. To take a specific example, if in the first determination in Table IV we increase R or R'' by 1/20 per cent, a change of -1/3 per cent is introduced in the value of L ; similarly changing R or R' by 1/20 per cent we alter L by +1/4 per cent. In regard to r , L varies only as the first power. Fortunately these errors in L introduced by variation of the resistance are not all of the same sign, and since the current through r must of necessity be quite small (about $\frac{1}{100}$ ampere), the currents through $R + R$ and $R' + R''$ are nearly equal. Hence if we assume an increase of all the resistances of 1/20 per cent, the value of L is increased by about one part in one thousand three hundred. Therefore, to obtain the best results with this method we must measure the resistances carefully and avoid much heatings. To avoid heating of the resistances the following plan of taking readings was adopted. The bridge was first adjusted, this being done by altering R' and R , until on reversing the current through the hanging coil of the electro-dynamometer there was zero deflection indicating a ninety degree phase difference between the fixed and suspended coils. By means of the switch S (vide fig. 5) the current was turned through the resistance X , made closely equal to the impedance of the network. This was done to avoid change in the frequency. After a considerable interval, sufficient to avoid the effect of any slight heating in the preliminary adjustment, the current was again switched through the network and R , remaining the same, R' readjusted to give zero deflection, the exact time of adjustment being noted in the chronograph sheet. This adjustment when made would usually hold good for several minutes. However, it was not usually necessary to run the current through the network longer than twenty or thirty seconds. This process was repeated a number of times with longer or shorter intervals and then the resistances R' and R , measured. The other arms of the bridge were measured once or twice during the evening; this time was chosen as being free from various disturbances. Generally for several successive observations the same value of R' would give zero deflection, R , remaining the same. The mean of the frequencies measured for the several observations was taken as being nearest correct.

If, however, R' had to be altered, the observations were regarded as separate and so worked out.

It was not the purpose of the authors to measure inductance to any great degree of accuracy, but to indicate the possibilities of the methods. Resistances were measured only to about two or three parts in ten thousand on the post office box, which had a considerable temperature correction. Still it will be seen that the determinations of L , except those of coil P, differ from the mean by less than 3/10 per cent.

The measurements of coil P were among the first made with this method, and the resistances were not measured with as much care as in the case of the other coils.

TABLE III.
Oct. 24, 1903.

R_s	R''	r	R_s	R'	n	L
103.75	103.67	425.06	900.76	870.9	65.644	.5722 henry
"	"	"	900.76	870.9	65.614	.5725 "
"	"	"	948.23	919.72	65.763	.5715 "
"	"	"	989.3	961.7	65.763	.5737 "
"	"	"	900.76	870.94	65.614	.5723 "
"	"	"	948.23	918.67	65.614	.5733 "
"	"	"	989.3	961.7	65.614	.5749 "
Mean						.57291 "

TABLE IV, Coil C.
Dec. 10, 1903.

Temp	R_s	R''	r	R'	n	L (henrys)
16°·3C.	99.907	99.78	968.8	1199.86	1088.6	63.288 1.3044

After the elapse of some minutes.

"	"	"	"	1199.86	1088.6	63.288 1.3044
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Dec. 13, 1903.

15°·0	99.94	99.42	968.9	1200.47	1088.0	62.366 1.3055
"	"	"	"	"	"	62.446 1.3008
"	"	"	"	"	"	62.259 1.3047
"	"	"	"	"	"	62.206 1.3058

Mean 1.30257

TABLE V, Coil S.
Nov. 13, 1903.

R_s	R''	r	R_s	R'	n	L (henrys)
100.0	100.0	425.06	1815.5	1767.2	65.393	1.0309
"	"	"	"	1766.67	65.393	1.0366
"	"	"	1862.9	1815.5	65.688	1.0303
"	"	"	1904.0	1856.96	66.091	1.0311
"	"	"	1904.0	1856.96	65.763	1.0353
Mean						1.0329

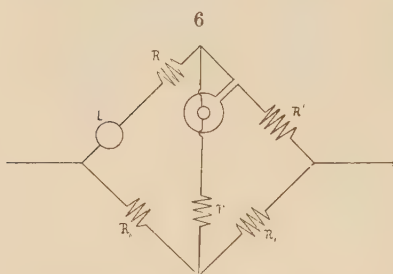
TABLE VI, Coil S.

May 13, 1904.

R_u	R''	r	R_i	R'	n	L	
100.16	99.42	1020.2	1816.9	1734.0	78.121	1.0345	henrys
"	"	"	"	1734.5	78.303	1.0359	"
"	"	"	"	1760.6	61.797	1.0307	"
"	"	"	"	1734.5	78.278	1.0325	"
"	"	"	1905.3	1819.3	81.410	1.0330	"
"	"	"	"	1818.3	81.700	1.0365	"
"	"	"	"	1848.8	62.604	1.0354	"
"	"	"	1616.0	1529.6	76.456	1.0333	"
"	"	"	"	1528.3	76.068	1.0294	"
"	"	"	"	1530.1	76.428	1.0299	"
						Mean 1.02331	"

III.

Method 13.—Method 13 was next tried. This is also an



absolute method for measuring either self-inductance or capacity. It is a zero deflection method depending on a ninety degree phase difference just as in method 14. In fact it differs from that method only in that the fixed coils of the electro-dynamometer are no longer in the battery arm of the bridge. The connections are shown in the diagram. Neglecting the self-inductance of the fixed and hanging coils of the electro-dynamometer, the formula for the method may be deduced as follows. Proceeding as before, we find the ratio of the current in the fixed and hanging coils

$$\frac{C_2}{C_1} e^{i(\phi_2 - \phi_1)} = \frac{r(R_u + R_i) + R_i(R_u + R'' + ipL)}{R'R_u - R_iR'' - ipR_iL}$$

Rationalizing the fraction, and taking the real parts, we have for zero deflection;

$$p^2 L^2 = \frac{\{R'R_u - R_iR''\} \{r(R_i + R_u) + R_i(R_u + R'')\}}{R_i^2}$$

We should expect this method not to be as sensitive as Method 14, for the reason that we must in general have a smaller current in the fixed coils of the dynamometer. Moreover, since the formula is similar in character to that of Method 14, we would

look for the value of L to be affected in the same way by a slight heating of one or more of the resistances. { The formula for Method 14, neglecting hanging coil correction is

$$p^2 L^2 = \frac{[R_1 R'' - R' R_2] [r(R' + R_1 + R_2 + R'') + (R' + R_1)(R'' + R_2)]}{R(r + R'' + R)} \}$$

The method was set up and some determinations made of the inductance of coil C. The method proved much less sensitive than Method 14, under comparable conditions only about one-half. By sensitive is meant the degree of accuracy to which it is possible to change the resistances, in securing adjustment.

It is to be noted that the formula as given neglects the self-inductance of the fixed and hanging coils of the electro-dynamo-meter. The exact formula is not difficult to work out, but the method is hardly sensitive enough to justify the trouble.

A few measurements of the inductance of coil C were made with some care. The values obtained were low compared with those previously obtained, as will be seen from the following example.

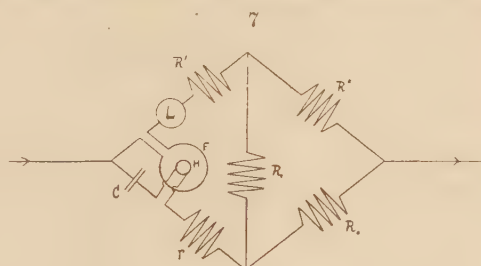
$$\begin{array}{lll} R_1 = 1201.7 \text{ ohms} & R'' = 1560.6 \text{ ohms} & r = 309.91 \text{ ohms} \\ R_2 = 99.89 \text{ " } & R' = 133.33 \text{ " } & n = 69.153 \text{ " } \\ & L = 1.1483 \text{ henrys} & \end{array}$$

IV.

Methods 1, 2, 3, etc.—Attention was not turned to the first six methods given in Professor Rowland's paper, which are for the comparison of self-inductance with capacity. They are all zero deflection methods depending on a ninety degree phase difference. As will be seen from the formulæ, the value of $\frac{L}{C}$ is independent of the frequency. Two of these methods have already been used.

Method 6 was studied by Mr. Penniman but did not yield favorable results. Method 3 was employed by Mr. Potts in the course of some work on electric absorption, and was found very satisfactory.

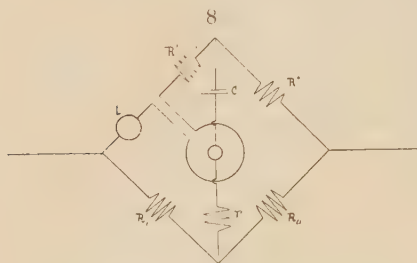
The diagrams and formulæ for the methods are as follows. The method of deducing the formulæ is the same as that followed in the preceding cases.



Method 1.—See fig. 7.

$$\frac{L}{C} = \frac{[r(R_i + R'') + R_u(r + R_i)] [R'(R_i + R_u) + R''(R_i + R')]}{(R_i + R'' + R_u)^2}$$

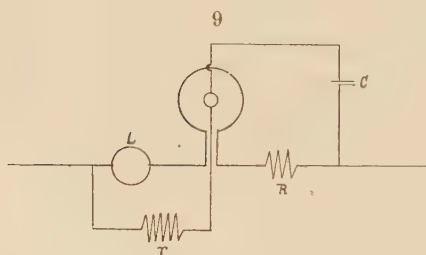
The trouble with this method was its lack of sensibility. Various values of resistances and coils were tried, but with the arrangement, which was as sensitive as possible, the resistances could not be adjusted closer than 1/5 per cent. This also when the maximum current was flowing through the network. It will be noted that the formula is quite similar to that of Method 14, so we should expect



the value of $\frac{L}{C}$ to be affected in the same way as was L by a slight change in one or more of the resistances. The values of the resistances which seemed to give the most sensitive arrangement when coil P was compared with a $\frac{1}{3}$ -microfarad mica condenser were,

$$\begin{aligned} R' &= 336 \text{ ohms} \\ R_i &= 1200 \text{ ohms} \end{aligned}$$

$$\begin{aligned} R'' &= 100 \text{ ohms} = R \\ r &= 3900 \text{ ohms} \end{aligned}$$



Method 2.—See fig. 8. When $L = .570$ and $C = \frac{1}{3}$ -microfarad. The values of the resistances which seemed to give the most sensitive arrangement were,

$$\begin{aligned} R'' &= R_u = 100 \text{ ohms} \\ R_i &= 1600 \text{ ohms} \\ R' &= \text{about } 1600 \text{ ohms} \end{aligned}$$

This method did not seem to be capable of adjustment much closer than 1 per cent even when the current through the network was .08 amp., which was much too large to avoid heating.

Method 3.—See fig. 9.

This is the simplest of the methods for determining $\frac{L}{C}$. The formula is

$$\frac{L}{C} = rR'.$$

As has already been stated, this method has been used with

success. The present writers set up the method and compared coil P with the $\frac{1}{3}$ -microfarad condenser. The method is very sensitive. With $r = 3070$ ohms and $R = 485$ ohms, a change of two ohms in r caused a change of 1^{mm} in the deflection of the electro-dynamometer. Determinations of the ratio $\frac{L}{C}$ were not made for the reason that if we wish accurate results it is necessary to measure the absorption resistance of the condenser. This should be done at each determination of $\frac{L}{C}$, as the absorption has been found to vary greatly both with the temperature and the frequency.

The diagrams and formulæ for Methods 4 and 5 are given: also the values of the resistances of the arms which seemed to give the most sensitive arrangement. As before, coil P, was compared with the $\frac{1}{3}$ -microfarad condenser.

Method 4.—See fig. 10.

$$\frac{L}{C} = \frac{[R'(r + R_u) + R''(R' + R_u)] [R'(R'' + R_u) + R''(R' + R'')]}{R'R''}$$

As adjusted,

$$R = 88.54 \text{ ohms}$$

$$R'' = 1021.8 \text{ ohms}$$

$$R' = 99.5 \text{ "}$$

$$r = 492.0 \text{ "}$$

$$R_u = 317.5 \text{ "}$$

With $C = .086$ ampere, a change of 4.4 ohms in r is barely detectable and a change of ten ohms in R' produces only 2^{mm} deflection as measured on the scale of the electro-dynamometer.

Method 5.—See fig. 11.

$$\frac{L}{C} = \frac{[R_u(R'' + R_u) + R_u(R'' + R')] [R'(R'' + R_u) + r(R' + R'')]}{(R' + R'')(R'' + R_u)}$$

As adjusted,

$$R = 100 \text{ ohms}$$

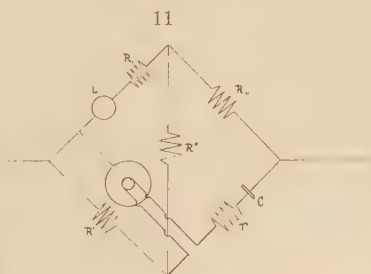
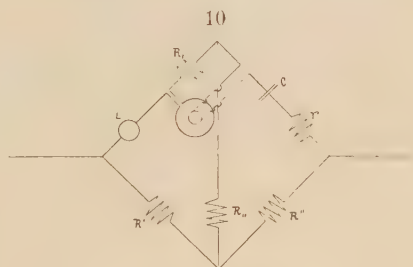
$$R'' = 88.5 \text{ "}$$

$$R' = 1041.5 \text{ "}$$

$$R_u = 317.5 \text{ "}$$

$$r = 1678.4 \text{ "}$$

This method is somewhat more sensitive than Method 4, however, it is not possible to adjust closer than one part in five hundred.



V.

Methods 7, 8, 9, 10, 11, and 12.

Of the remaining zero-deflection methods which depend on a ninety degree phase difference, little needs to be said. Methods 7 and 8 are for determining $\frac{L}{C}$ but involve mutual inductance and are intended to be used with doubly wound coils. For accuracy greater than one per cent such coils are very undesirable on account of the electrostatic capacity, and should not be used. Methods 9 and 10 are for comparing mutual inductance with capacity. Method 12 was investigated by Mr. Penniman.

VI.

Methods 15-24.

The rest of the methods require at least two adjustments; first, the Wheatstone bridge must be balanced with direct currents, then adjustment made with alternating current so that the current through the bridge arm is zero. Of these methods, 15 and 16 are for comparing two inductances, two capacities or inductance with capacity. However, Method 15 requires after balancing the bridge with direct currents two simultaneous adjustments with alternating current; and Method 16 requires many resistances of known ratio. Method 17 is for the measure-

ment of $\frac{L}{M}$, but as Professor Rowland points out: "It is difficult to apply, as two resistances must be adjusted and the adjustment will only hold while the current period remains constant." These methods did not seem to be worthy of trial.

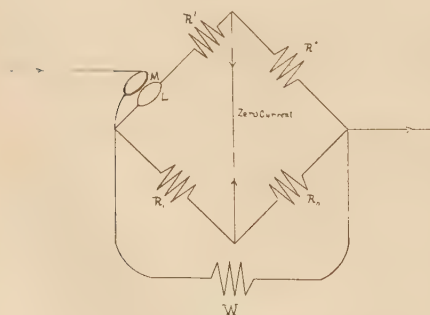
Method 18 is for the determination of $\frac{L}{M}$ where L and M belong to the same coil. This method is simpler than

most of the others and seemed rather more promising, so it was given a trial. The diagram of the method is shown in fig. 12.

The bridge is first balanced with direct currents giving

$$R'R_{\parallel} = R'R''.$$

12



Then with alternating current W is adjusted until there is zero current through the bridge arm, as shown by an electro-dynamometer or telephone. When this is the case the mutual inductance completely counterbalances the self-inductance, so that the currents and electromotive forces are in phase in each arm of the bridge. Representing current through R' by I' , etc.,

$$R_i I_i = R' I' + ipLI' - ipM (I' + I_i + I_w)$$

also

$$R_i I_i = R' I'$$

and

$$\frac{I'}{I_w} = \frac{W}{R' + R''}$$

$$\frac{pM (I' + I_i + I_w)}{I'} = pL$$

Hence

$$\frac{L}{M} = 1 + \frac{R'}{R_i} + \frac{R' + R''}{W}$$

The method was set up so that by means of switches the resistance W could be replaced by the battery, and the electro-dynamometer by the galvanometer. The method proved very unsatisfactory. Using the electro-dynamometer in the bridge arm, its coils being connected in series, W could not be adjusted closer than three per cent. Different values of the resistance were tried but the result was about the same. The electro-dynamometer was then replaced by a telephone. It was then possible to adjust to about two per cent but the noise of the dynamo interfered somewhat with the use of the telephone. The conclusion was that the method is not sufficiently sensitive to be of value.

Method 19 is also intended for the measurement of $\frac{L}{M}$ but involves the use of a doubly wound coil.

Methods 20 and 21 require a specially doubly wound inductance coil. They were not thought sufficiently promising to merit the construction of such a coil.

Method 22 is Carey Foster's method adapted to alternating currents. This method was rather thoroughly investigated by Heydweiller.*

Method 23 requires two simultaneous adjustments with alternating currents, and Method 24 is only of special use in comparing two doubly wound coils.

Summary.

With the exception of Methods 14, 3 and 25 the Rowland methods tested are of little value, their chief defect being a

* Wied. Ann., liii, p. 499, 1894.

lack of sensibility. Method 14, or more properly, the Oberdeck method, is without doubt the best method that we have for the absolute measurement of self-inductance in terms of resistance. It is easy to apply and if proper care be exercised results should not differ from the mean by more than 2/10 per cent. Greater accuracy than this is undoubtedly attainable and ought not to be a matter of any great difficulty, for in the work here described the frequency determination could have been improved and the resistances much more accurately measured. Method 25 is better than the Rayleigh method as ordinarily used, is very easy to work and has a very simple formula, involving little calculation. Its accuracy is probably limited to about one per cent. Method 3 for comparing self-inductance with capacity is very sensitive and was found by Dr. Potts to be very satisfactory.

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